

Finite Element Model Calibration of Sandwich Structure Based on Mixed Numerical Experimental Technique

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Abstract. This paper presents the implementation of the mixed numerical experimental technique for the finite element (FE) model calibration of a sandwich structure by using modal data. Model calibration is conducted by minimizing the difference between the numerical and experimental dynamic parameters. In this study, instead of the direct minimisation of the response discrepancy, the experiment design and the response surface method is employed to solve the inverse (calibration) problem. Numerical eigenfrequencies are obtained by performing FE calculations in sample points derived by Latin Hypercube experiment design where each sample point represents a unique engineering constant configuration in the FE model of the sandwich panel. Using the information on the dynamic responses of the panel in the sample points, response functions describing the relationship between the engineering constants and the calculated eigenfrequencies are obtained by means of response surface method. Genetic algorithm is employed to solve the minimisation of the response discrepancy where the response functions instead of FE calculations are used to obtain the numerical modal frequencies. The verification results show that proposed method is capable to calibrate a numerical model with a good prediction accuracy and small uncertainties.

1. Introduction

Due to widespread use of finite element (FE) method for structural analysis of complex engineering structures, development of numerical model calibration methods have attracted a lot of attention over the last decades. Accurate FE model that represents relevant physical processes of the actual structure may be used for simulation and prediction of structural behaviour under different loading conditions. However, initial FE models are inevitably corrupted by uncertainties in geometry, material properties and boundary conditions, which may then lead to simulation results of structural behaviour far from the actual ones. Thus, the initial model ought to be calibrated in terms of features of interest according to the physical model. Typically, calibration of FE model, also known as FE model updating, is a procedure that minimizes the differences between numerical and experimental data. Among different experimental testing available for engineering structures, modal analysis has been the most preferred approach. Development of numerical model calibration methods based on dynamic parameters (e.g. eigenfrequencies, mode shapes, damping ratios) has been reported in many papers [1-16].

Dynamic model calibration requires minimization of an objective function based on the difference between the numerical calculated and experimentally measured eigenfrequencies, while mode shapes are compared with various metrics such as the orthogonality and cross-orthogonality of the modes with respect to the mass distribution quantified by a FEM mass matrix [2]. FE model calibration methodologies in structural dynamics can generally be divided in two distinct categories: the direct methods [3] and the iterative methods [4]. The first approach directly updates the mass and stiffness matrices of the structure but it is very difficult to relate the changes inside the updated system matrices to physical properties of the FE model [1]. Furthermore, these methods can be very complicated for large structures with a detailed FE model and can result in ill-conditioned problems [5, 6]. Contrary, the iterative methods are more flexible and efficient for large and complex structures with detailed FE models as these methods update the physical properties of the FE model, such as material and geometric properties [7, 8]. The calibration process is typically characterised by defining calibration parameters and performing a large-scale optimization or manual analysis by trial and error. In this context, computational efficiency must be considered when model calibration is applied for complex structures with detailed FE model. Significant reduction in computational efforts can be achieved by using machine learning methods in the process of model calibration as it is shown by several authors [9-11]. Response Surface Method (RSM), Artificial Neural Network (ANN) and Multivariate Relevant Vector Machines (MRVM) among other traditional machine learning methods have been successfully employed to build the response models to calculate the modal parameters of a structure instead of FE calculations during the optimizing search process. In recent years, FE model calibration methods have been used for broad range of engineering structures such as bridges, dams, viaducts, precast structures, communications satellite [11-16].

This paper presents the implementation of the Mixed Numerical Experimental Technique (MNET) for the FE model calibration of a sandwich panel using modal data. The structure under investigation is a sandwich panel composed of two laminated carbon fibre reinforced plastic (CFRP) face sheets and aluminium alloy pyramidal truss core. The finite element model of the sandwich panel is built by using the FE software ABAQUS. Four independent engineering constants of a single transversally isotropic layer of CFRP face sheets and two engineering constants of the aluminium core are calibrated in order to improve correlation between the measured and calculated modal parameters.

2. Mixed Numerical-Experimental Technique

The model calibration approach used in this paper is based on the Mixed Numerical Experimental Technique. The general idea of MNET is that instead of measuring the property of interest, indirect procedures measure a number of related quantities and derive the unknown property from the experimental values of these quantities. A numerical model is used to relate the physical property of interest to the measured quantities. Then, the inverse problem is employed to derive a number of model parameters from the system's response to a particular input. The inverse problem is solved by minimizing the difference between the numerical model response and the experimentally measured one.

In the process of optimization search, direct optimisation algorithms require a large amount of iteration steps, and for each step multiple iterated finite element solutions are performed. This may lead to high computational costs when model calibration is applied for large scale and complex structures. In order to improve computational efficiency, in this paper instead of the direct minimisation of the response discrepancy, the method of experiment design and the response surface approach are proposed to solve the inverse problem. Numerical simulations of a structure are performed only in the sample points of the experiment design. The response surface approximations are obtained by using information on the dynamic responses of a structure in sample points. Now, the response functions instead of FE calculations are used to obtain the numerical modal frequencies used in the error functional. By this way, a significant reduction in the computational efforts can be

achieved compared to the conventional methods of optimisation. Flowchart of the proposed MNET approach is presented in figure 1.

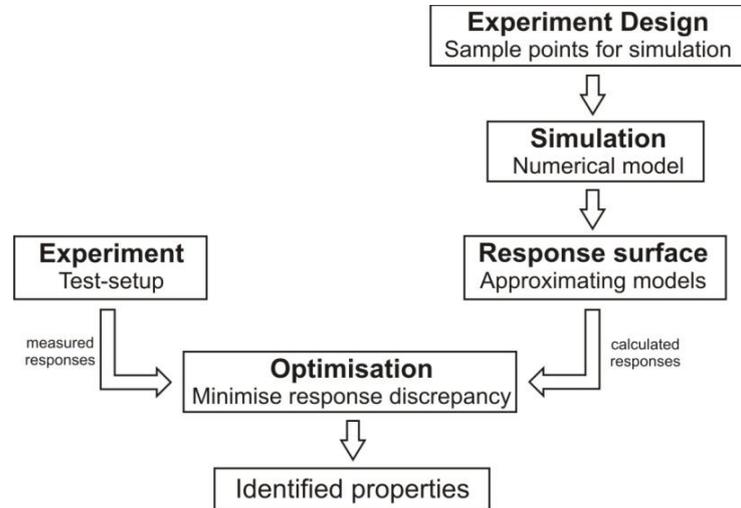


Figure 1. The flowchart of the MNET based on the experiment design and response surface method.

2.1. Parameters for FE model calibration

In this study, the proposed MNET approach is used for the finite element model calibration of a sandwich panel composed of two laminated carbon fibre reinforced plastic (CFRP) face sheets and aluminium alloy PA6 pyramidal truss core. The panel of length $l = 380.3$ mm, width $a = 50$ mm, and height $h = 27.8$ mm is considered. A global Cartesian co-ordinate system (x,y,z) was located in the corner of the panel, with z axis upward and normal to the face layer (figure 2a). The face sheets were cut out of a long unidirectional CRFP tape which were manufactured by means of a pultrusion process. The face sheets were composed of a single layer having material density $\rho = 1540$ [kg/m^3] and the total thickness $t = 1.4$ mm. For the CRFP laminated tape a local co-ordinate system $(1,2,3)$ was defined with the direction 1 along the fibres of the tape, 2 transverse to this direction, and 3 - through the thickness direction. A lamination angle $(\Phi = 0)$ of the fibers was defined between the x -axis and the 1 - axis (figure 2c). The pyramidal truss core was assembled from separate pieces which were cut out of an aluminium plate by means of water-jet cutting. The pieces were bonded together with a thermoset epoxy adhesive to form a continuous pyramidal truss core. The geometry of a single cell of the truss is given on figure 2b. The material density of the PA6 alloy was $\rho = 2700$ [kg/m^3]. The parent components were bonded together to form a sandwich panel with the same thermoset epoxy adhesive as used for the core assembly. The epoxy adhesive was applied locally on the face sheets where the contact with the core existed.

Proper selection of calibration parameters is crucial for the successful FE model calibration. Changes in the selected parameters should potentially have a considerable effect on the system's response. Modal characteristics of the structure are very sensitive to changes in stiffness and mass properties of the structure. Since the mass density of constitutive components of the sandwich panel have been determined with high accuracy using conventional testing methods, then the material properties of the structural components are selected for FE model calibration. The parameters to be calibrated are four engineering constants of a single transversally isotropic layer of CFRP face sheets and two engineering constants of the aluminium core:

$$E_1, E_2 = E_3 - \text{longitudinal and transverse Young's modulus of CFRP;} \\ G_{12} = G_{13} - \text{in-plane shear modulus of CFRP;}$$

$\nu_{12}=\nu_{13}$ - in-plane Poisson's ratio of CFRP;
 E - Young's modulus of aluminium;
 ν - Poisson's ratio of aluminium;

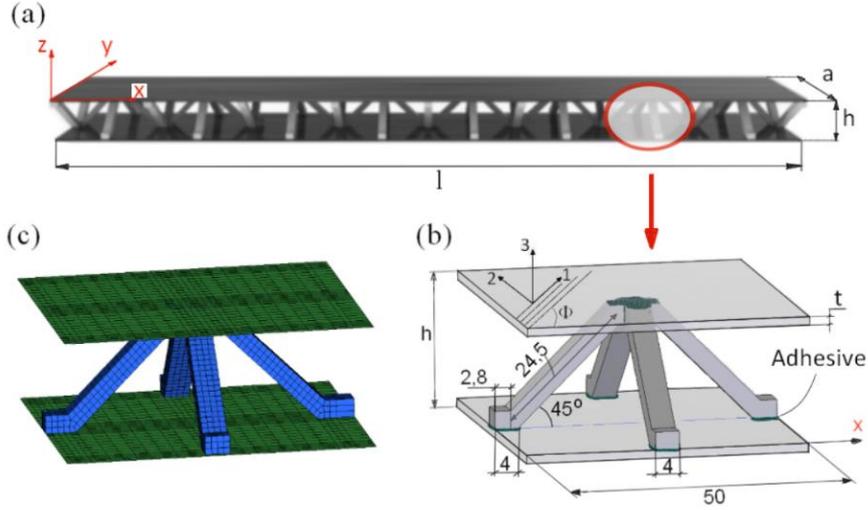


Figure 2: Sandwich panel with pyramidal truss core: (a) Manufactured sample; (b) Unit cell of the truss core sandwich panel; (c) Finite element model of the unit cell - dimensions in [mm].

2.2. Experiment design

Any experiment can be defined as series of tests on a system that is performed in order to study the relationship between the input variables and the output responses of the process [17]. In this context, the objectives of any experiment may be outlined as following: determining which of the input variables x is most influential on the response y ; determining where to set the influential x 's so that y is almost always near the desired nominal value and its variability is small. To fulfil these objectives there is an obvious need for an experimentation strategy or an approach for design of experiments.

In this study, a minimal Mean Squared Distance (MSD) Latin Hypercube experiment design is employed [18]. This experiment design is space filling design, which gives minimal MSD between points in design space R and nearest point from the experiment design D

$$MSD = \sqrt{\left(\frac{1}{n}\right) \sum_{v=1}^n \min_{u=1, \dots, N} \left[\sum_{i=1}^m (w_{iv} - x_{iu})^2 \right]} \quad (1)$$

where w_v is a large sample from the points in design space R ($v=1, \dots, n$). This design gives points uniformly distributed in design space and tends to minimise the expected mean squared error of local quadratic approximation. The experiment design with 6 variables and 101 sample points was constructed. The initial model data of the sandwich panel was obtained from material documentation and static testing data. The upper and lower bounds of engineering constants for the sandwich panel are given in table 1. Each sample point represents a unique engineering constant configuration and for each sample point the finite element solution of the eigenvalue problem was performed.

Table 1. Domain of interest for engineering constants for the sandwich panel

Engineering constant	E_1 , GPa	E_2 , GPa	G_{12} , GPa	ν_{12}	E , GPa	ν
min	120.8	8.9	6.6	0.32	56.2	0.21

max	163.5	12.1	8.9	0.44	76.2	0.29
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2.3. Finite element model

The Finite Element Method was used to develop the numerical model of the sandwich panel (figure 2b). The FE model of the laminated face layers based on the First Order Shear Deformation Theory (FSDT). Shell layered linear elastic elements S4R were used to model the laminated face sheets. C3D8R three dimensional linear elastic elements were used to develop the FE model of the aluminium truss core. The face layers and truss core elements were connected together by node fit at the locations where the contact spots between the elements existed. The eigenvalue problem for undamped free vibrations of the sandwich panel was represented by $(\mathbf{K} - \omega_n^2 \mathbf{M})\Psi_n$, where \mathbf{K} and \mathbf{M} are the global stiffness and mass matrices of sandwich panel, respectively; Ψ_n are the eigenvectors (mode shapes) corresponding to eigenvalues $\omega = 2\pi f_n$, with f_n being eigenfrequencies. The Simulia/Abaqus software was used to solve the eigenvalue problem. The Frequency analysis with the Lanczos eigensolver was applied to determine eigenvalues and corresponding eigenvectors of the undamped vibrations of the FFFF sandwich panel (all edges free).

2.4. Response surface method

The information about modal response of the present sandwich panel finite element model can be represented as a data table, where the response functions (numerically calculated eigenfrequencies) $y(\mathbf{x})$ is in relationship with the six calibration variables \mathbf{x} representing the engineering constants. The goal is to build the relation $y(\mathbf{x})$ in the mathematical form or so called equation of regression by using the data obtained from finite element simulations in the sample points of the experiment design. The most popular approximating models are polynomial models created by performing a least squares curve fit to a set of data. For example, the second-order polynomial can be expressed as follows:

$$y(\mathbf{x}) = b_0 + \sum_{i=1}^m b_i x_i + \sum_{i=1}^m \sum_{j=i}^m b_{ij} x_i x_j \quad (2)$$

where m is a total amount of variables; b_0 , b_i , b_{ij} is unknown coefficients of regression functions. There are two requirements for the equation of regression - accuracy and reliability. Accuracy is characterized by a minimum of standard deviation. Increasing the number of terms in the equation of regression it is possible to obtain a complete agreement between the experimental data and the values given by the equation of regression. Reliability of the equation of regression means that accuracy for the sample points and for any other point in the domain of interest are approximately the same. If the number of terms of the equation of regression decreases, the reliability of the model increases. A compromise between the accuracy and reliability of the model must be found [19].

In this study, the second-order polynomial was found to give the best agreement between the numerical simulation data and the values given by the equation of regression. For the illustration of the method, response surface graphs representing relationship between different engineering constants and calculated eigenfrequencies of the panel are presented in figure 3. Similar response surfaces were built for all eigenfrequencies.

2.5. Vibration experiment

The eigenfrequencies and corresponding mode shapes of the sandwich panel were measured using a POLYTEC PSV-400-B Scanning Laser Vibrometer. Equipment consists of a PSV-I-400 LR optical scanning head equipped with high sensitivity vibrometer sensor (OFV-505), an OFV-5000 controller, PSV-E-400 junction box, an amplifier Bruel&Kjaer type 2732, and a computer system with data

acquisition board and PSV Software. In order to exclude the influence of boundary conditions on the calibration results, the sandwich panel was hung using two thin treads bonded to top corners of the panel, thus simulating free boundary conditions (figure 4).

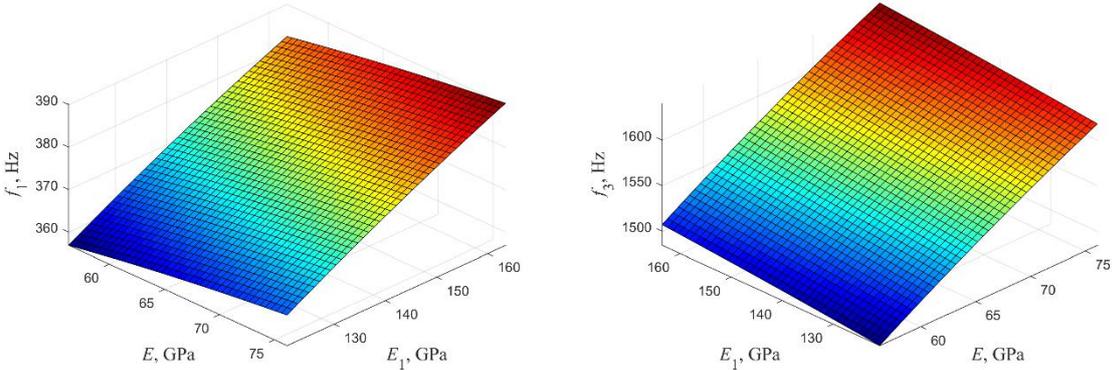


Figure 3. Relationships between the engineering constants and calculated eigenfrequencies.

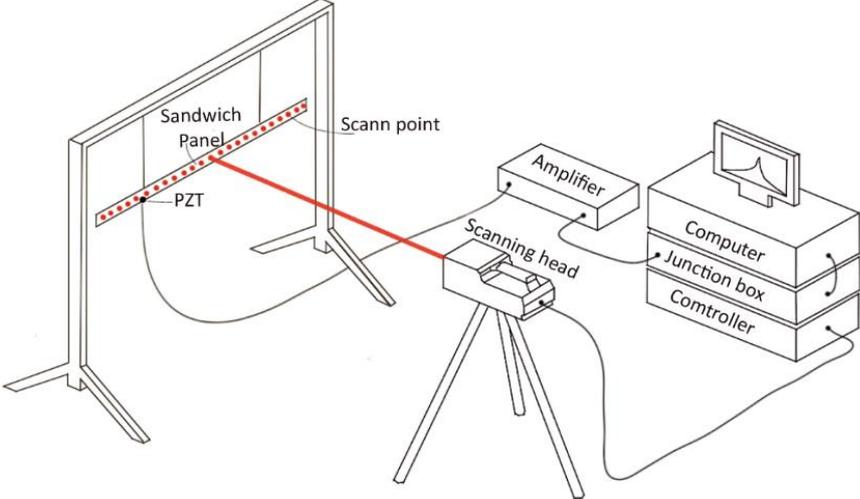


Figure 4. Experiment set-up for vibration testing.

The panel is excited by a periodic input chirp signal generated by the internal generator with a 2400Hz bandwidth through a piezoelectric actuator. Response of the panel was measured by the scanning laser and stored on a PC. The modal frequencies and corresponding mode shapes (figure 5) are obtained by taking the fast Fourier transform of the response signal [20, 21].

2.6. Error functional and minimisation problem

Numerical model calibration of the sandwich panel is achieved through minimisation of an error functional that expresses the relative difference between the measured and numerically calculated eigenfrequencies (RSM models)

$$\Phi(\mathbf{x}) = \sum_{i=1}^I w_i \frac{\left(f_{i_{EXP}}^2 - [f_{i_{FEM}}(\mathbf{x})]^2 \right)^2}{f_{i_{EXP}}^4} \quad (3)$$

Here w_i are weights for the error functional and I is the number of used eigenfrequencies. Weights are equal to 1 for eigenfrequencies used in the error functional and equal to 0 for the unused ones. Minimisation of the error functional is subjected to upper and lower bounds of the calibration parameters:

$$x_i^{\min} \leq x_i \leq x_i^{\max}; \quad i = 1 \dots 6 \quad (4)$$

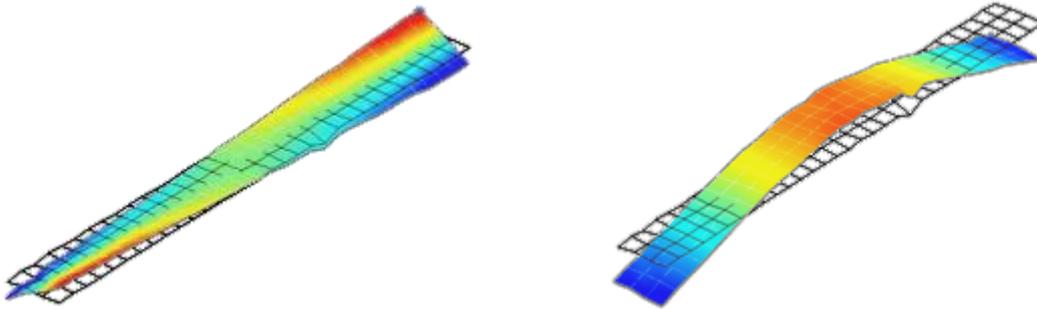
3. Model calibration results

Minimisation of the error functional (3) subjected to the constraints (4) was performed by employing the different optimisation algorithms such as the simulated annealing (SA) algorithm, genetic algorithms (GA), direct search (DS) algorithms, and particle swarm (PS) algorithm. Finally, the GA was selected in this study for its ability to reach the true global minimum.

By minimising the functional, calibration parameters \mathbf{x} were obtained (table 2). It must be noted that eigenfrequency no. 3 was excluded from the error functional in the last iteration of the minimisation. Verification of the obtained results was performed by comparing the experimentally measured eigenfrequencies with the numerically calculated ones using the calibrated engineering constants (\mathbf{x}^*). Residuals were calculated by the following expression:

$$\Delta_i = \left| \frac{f_{i_{FEM}}(\mathbf{x}^*) - f_{i_{EXP}}}{f_{i_{EXP}}} \right| \times 100 \quad (5)$$

As it is seen in table 3, in general a good agreement between the experimental and numerical eigenfrequencies is achieved. Residuals for frequencies do not exceed 4% indicating a successful model calibration.



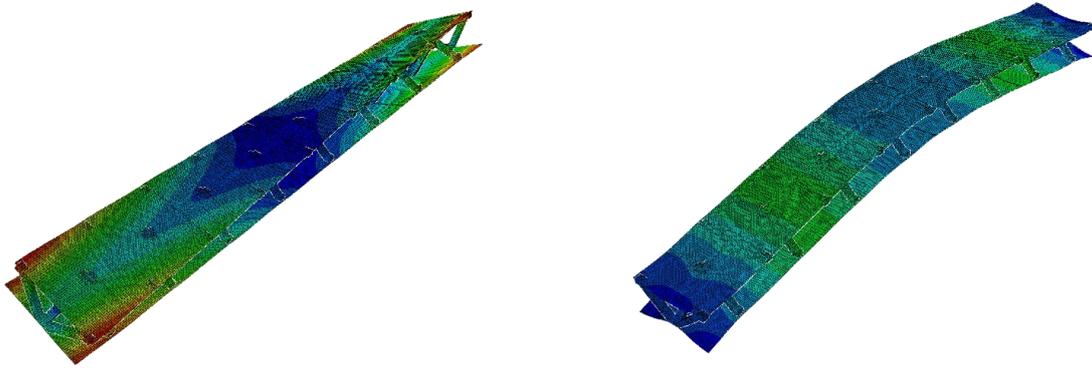


Figure 5. Examples of mode shapes: top: experimental; bottom: numerical; left: Mode 1; right: Mode 2.

Table 2. Calibrated engineering constants for the sandwich panel.

Engineering constant	E_1 , GPa	E_2 , GPa	G_{12} , GPa	ν_{12}	E , GPa	ν
Calibrated value	133.5	11.9	7.3	0.33	75.4	0.24

Table 3. Experimental and numerical frequencies, and corresponding residuals.

Mode No.	1	2	3	4	5
Experimental, Hz	382.62	1257.5	1482.0	2079.9	2128.7
Numerical, Hz	382.57	1250.4	1536.7	2116.8	2121.5
Δ_i , %	-0.01	-0.56	3.69	1.78	-0.34

4. Conclusions

In this paper, the finite element model calibration of a pyramidal truss core sandwich panel was conducted by using modal data. Objective of the dynamic model calibration is minimization of an error function representing the difference between the numerical calculated and experimentally measured eigenfrequencies. The experimental eigenfrequencies and mode shapes of the sandwich panel were measured by using a scanning laser vibrometer, while corresponding numerical data was calculated by FE software ABAQUS. In order to improve computational efficiency, the method of experiment design and the response surface approach were proposed to solve the inverse problem. The main advantage of the proposed method is a significant reduction of the computational efforts in comparison to the conventional model calibration methods based on direct minimisation of response discrepancy. The verification results show that proposed method is capable to calibrate a nonlinear model with a good prediction accuracy and small uncertainties.

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